Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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7	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MS03

Unit Statistics 3

Tuesday 22 June 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions in the spaces provided.

1 Ffion, as part of her research project, measured the stem length and the cap diameter of each of a random sample of 24 matsutake mushrooms. Using these measurements, she calculated the value of the product moment correlation coefficient to be 0.336, correct to three significant figures.

Assuming that her measurements came from a bivariate normal distribution, test, at the 5% level of significance, the hypothesis that there is no correlation between the stem length and the cap diameter of matsutake mushrooms. *(4 marks)*

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2 Rodney and Derrick, two independent fruit and vegetable market stallholders, sell punnets of locally-grown raspberries from their stalls during June and July.

The following information, based on independent random samples, was collected as part of an investigation by Trading Standards Officers.

			Weight of raspberries in a punnet (grams)	
		Sample size	**Sample mean**	**Sample standard deviation, *s***
Stallholder	**Rodney**	50	225	5
	Derrick	75	219	8

(a) Construct a 99% confidence interval for the difference between the mean weight of raspberries in a punnet sold by Rodney and the mean weight of raspberries in a punnet sold by Derrick. *(5 marks)*

(b) What can be concluded from your confidence interval? *(2 marks)*

(c) In addition to weight, state one other factor that may influence whether customers buy raspberries from Rodney or from Derrick. *(1 mark)*

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3 The weekly number of hits, S, on Sam's website may be modelled by a Poisson distribution with parameter λ_S. The weekly number of hits, T, on Tina's website may be modelled by a Poisson distribution with parameter λ_T.

During a period of 40 weeks, the number of hits on Sam's website was 940.

During a period of 60 weeks, the number of hits on Tina's website was 1560.

Assuming that S and T are independent random variables, investigate, at the 2% level of significance, Tina's claim that the mean weekly number of hits on her website is greater than that on Sam's website. *(7 marks)*

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4 It is proposed to introduce, for all males at age 60, screening tests, A and B, for a certain disease.

Test B is administered only when the result of Test A is inconclusive.

It is known that 10% of 60-year-old men suffer from the disease.

For those 60-year-old men suffering from the disease:

- Test A is known to give a positive result, indicating a presence of the disease, in 90% of cases, a negative result in 2% of cases and a requirement for the administration of Test B in 8% of cases;
- Test B is known to give a positive result in 98% of cases and a negative result in 2% of cases.

For those 60-year-old men not suffering from the disease:

- Test A is known to give a positive result in 1% of cases, a negative result in 80% of cases and a requirement for the administration of Test B in 19% of cases;
- Test B is known to give a positive result in 1% of cases and a negative result in 99% of cases.

(a) Draw a tree diagram to represent the above information. *(4 marks)*

(b) (i) Hence, or otherwise, determine the probability that:

(A) a 60-year-old man, suffering from the disease, tests negative;

(B) a 60-year-old man, not suffering from the disease, tests positive. *(2 marks)*

(ii) A random sample of ten thousand 60-year-old men is given the screening tests. Calculate, to the nearest 10, the number who you would expect to be given an **incorrect** diagnosis. *(2 marks)*

(c) Determine the probability that:

(i) a 60-year-old man suffers from the disease given that the tests provide a positive result;

(ii) a 60-year-old man does not suffer from the disease given that the tests provide a negative result. *(5 marks)*

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5 In the manufacture of desk drawer fronts, a machine cuts sheets of veneered chipboard into rectangular pieces of width W millimetres and height H millimetres. The 4 edges of each of these pieces are then covered with matching veneered tape.

The distributions of W and H are such that

$\mathrm{E}(W) = 350 \qquad \mathrm{Var}(W) = 5 \qquad \mathrm{E}(H) = 210 \qquad \mathrm{Var}(H) = 4 \qquad \rho_{WH} = 0.75$

(a) Calculate the mean and the variance of the length of tape, $T = 2W + 2H$, needed for the edges of a drawer front. *(5 marks)*

(b) A desk has 4 such drawers whose sizes may be assumed to be independent.

Given that T may be assumed to be normally distributed, determine the probability that the total length of tape needed for the edges of the desk's 4 drawer fronts does not exceed 4.5 metres. *(5 marks)*

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6 (a) A district council claimed that more than 80 per cent of the complaints that it received about the delivery of its services were answered to the satisfaction of complainants before reaching formal status.

An analysis of a random sample of 175 complaints revealed that 28 reached formal status.

(i) Construct an approximate 95% confidence interval for the proportion of complaints that reach formal status. *(5 marks)*

(ii) Hence comment on the council's claim. *(2 marks)*

(b) The district council also claimed that less than 40 per cent of all formal complaints were due to a failing in the delivery of its services.

An analysis of the 50 formal complaints received during 2007/08 showed that 16 were due to a failing in the delivery of its services.

(i) Using an exact test, investigate the council's claim at the 10% level of significance. The 50 formal complaints received during 2007/08 may be assumed to be a random sample. *(5 marks)*

(ii) Determine the critical value for your test in part **(b)(i)**. *(2 marks)*

(iii) In fact, only 25 per cent of all formal complaints were due to a failing in the delivery of the council's services.

Determine the probability of a Type II error for a test of the council's claim at the 10% level of significance and based on the analysis of a random sample of 50 formal complaints. *(4 marks)*

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7 The random variable X has a Poisson distribution with parameter λ.

(a) (i) Prove, from first principles, that $\mathrm{E}(X) = \lambda$. *(3 marks)*

(ii) Hence, given that $\mathrm{E}(X(X-1)) = \lambda^2$, find, in terms of λ, an expression for $\mathrm{Var}(X)$. *(2 marks)*

(b) The mode, m, of X is such that

$$\mathrm{P}(X = m) \geqslant \mathrm{P}(X = m-1) \quad \text{and} \quad \mathrm{P}(X = m) \geqslant \mathrm{P}(X = m+1)$$

(i) Show that $\lambda - 1 \leqslant m \leqslant \lambda$. *(3 marks)*

(ii) Given that $\lambda = 4.9$, determine $\mathrm{P}(X = m)$. *(2 marks)*

(c) The random variable Y has a Poisson distribution with mode d and standard deviation 15.5.

Use a distributional approximation to estimate $\mathrm{P}(Y \geqslant d)$. *(5 marks)*

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END OF QUESTIONS

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